# Point spread function field super-resolution and interpolation

IAP, 05/06/2015

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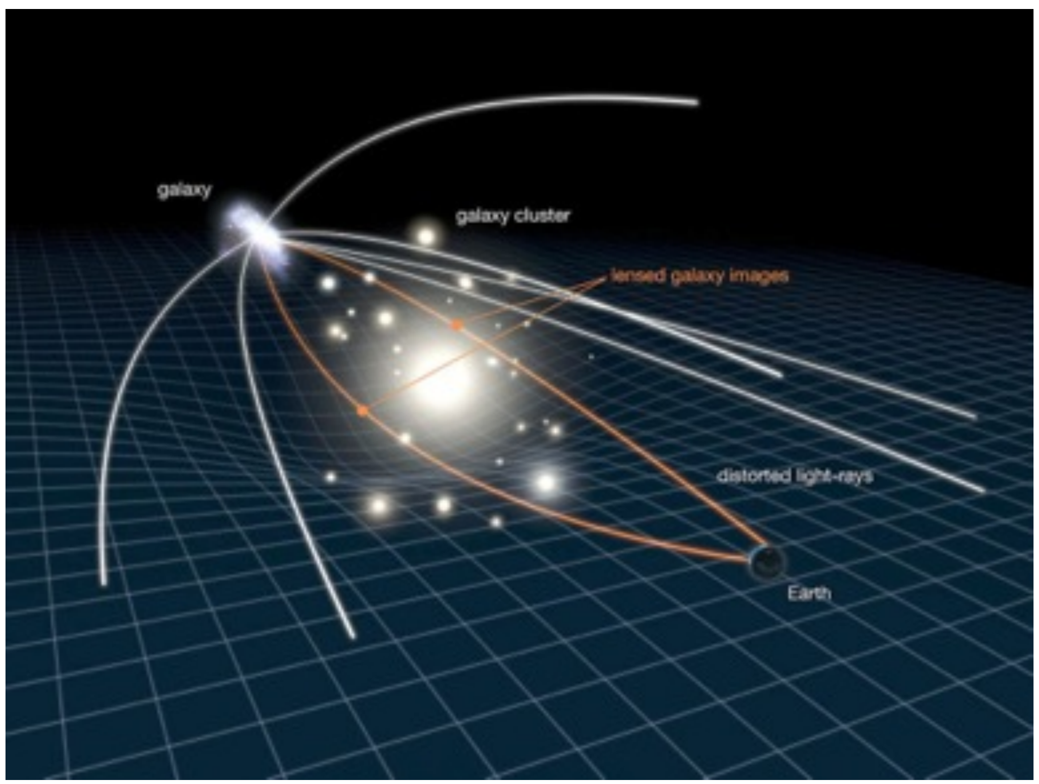
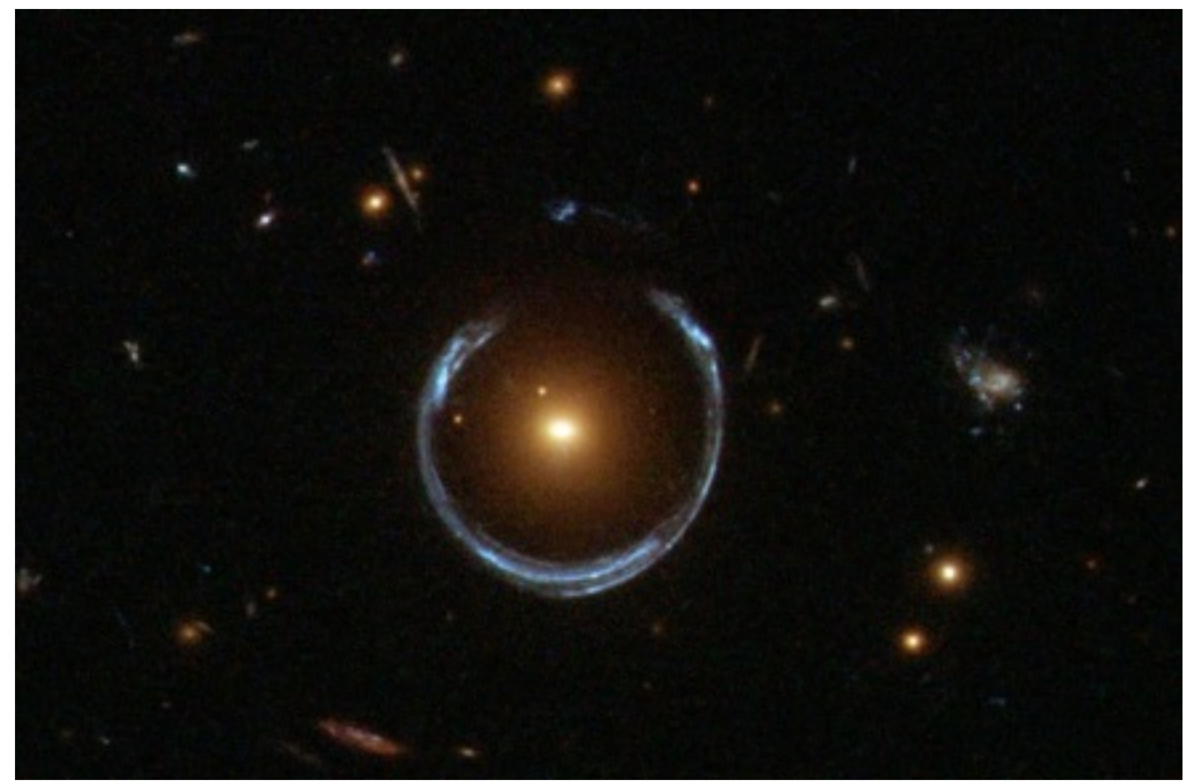
# Presentation Layout

- Scientific context
- Data analysis challenges
- PSFs super-resolution
- PSF interpolation/Optimal transport
- Conclusions

# Context

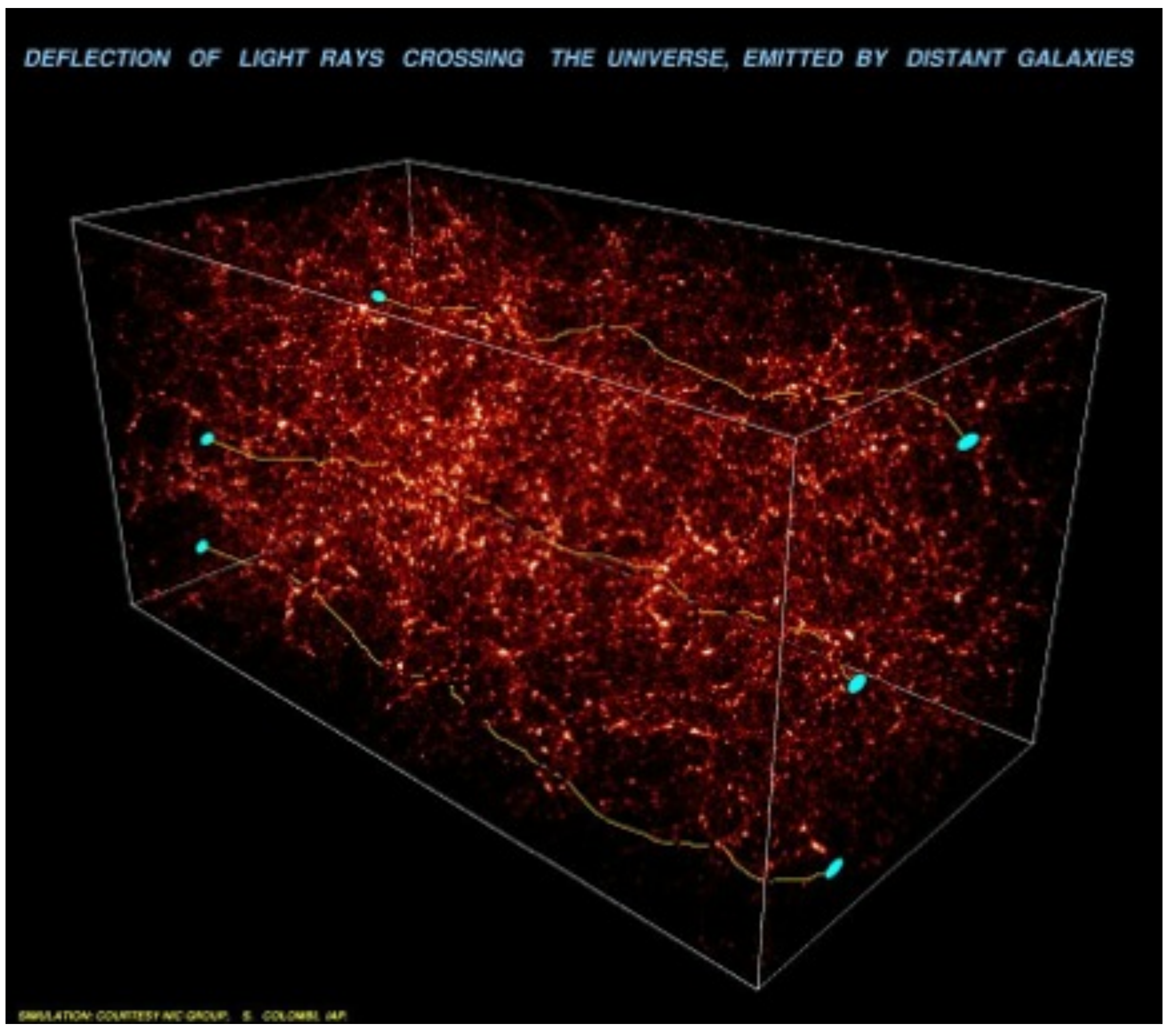
Gravitational lensing

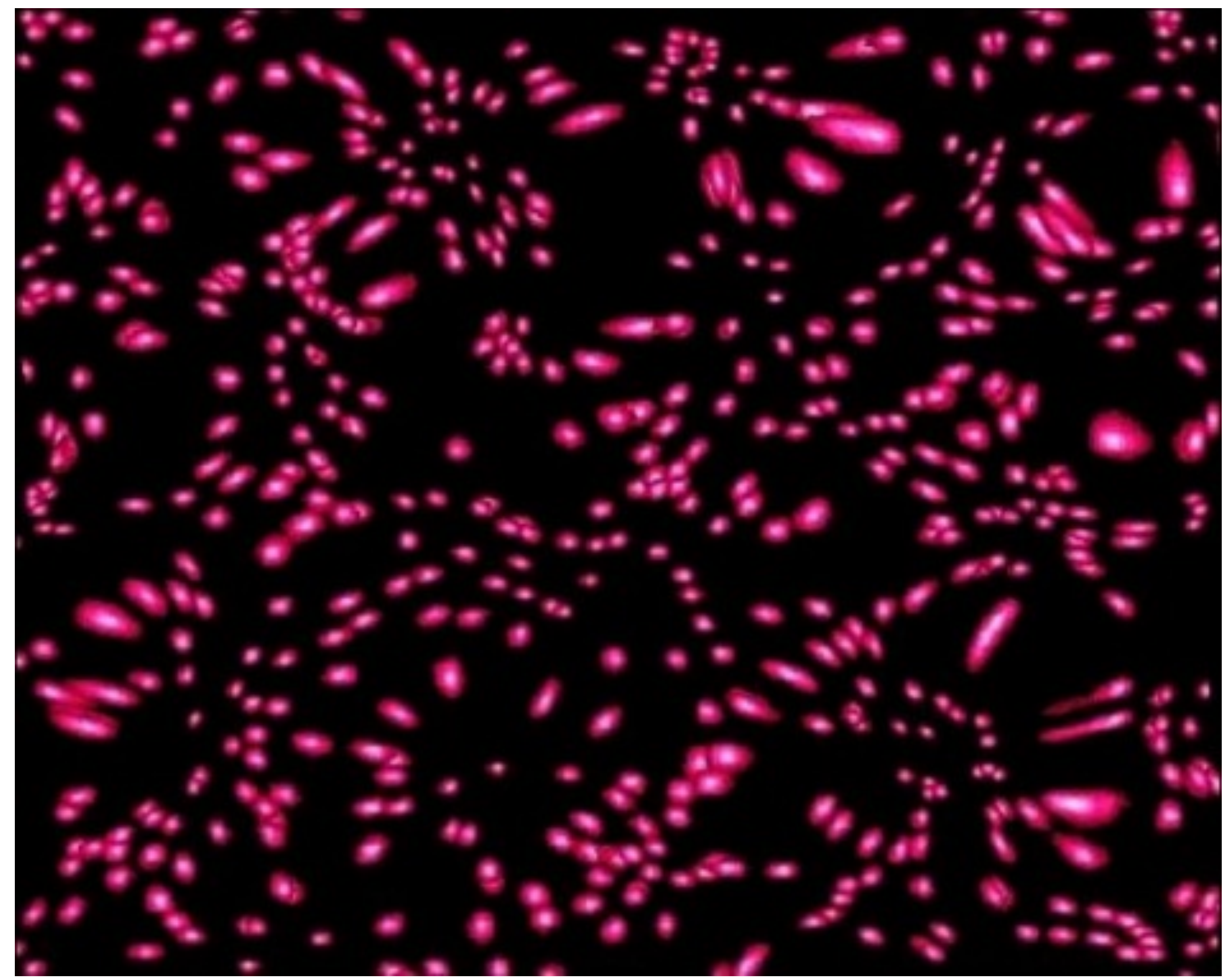


# Context

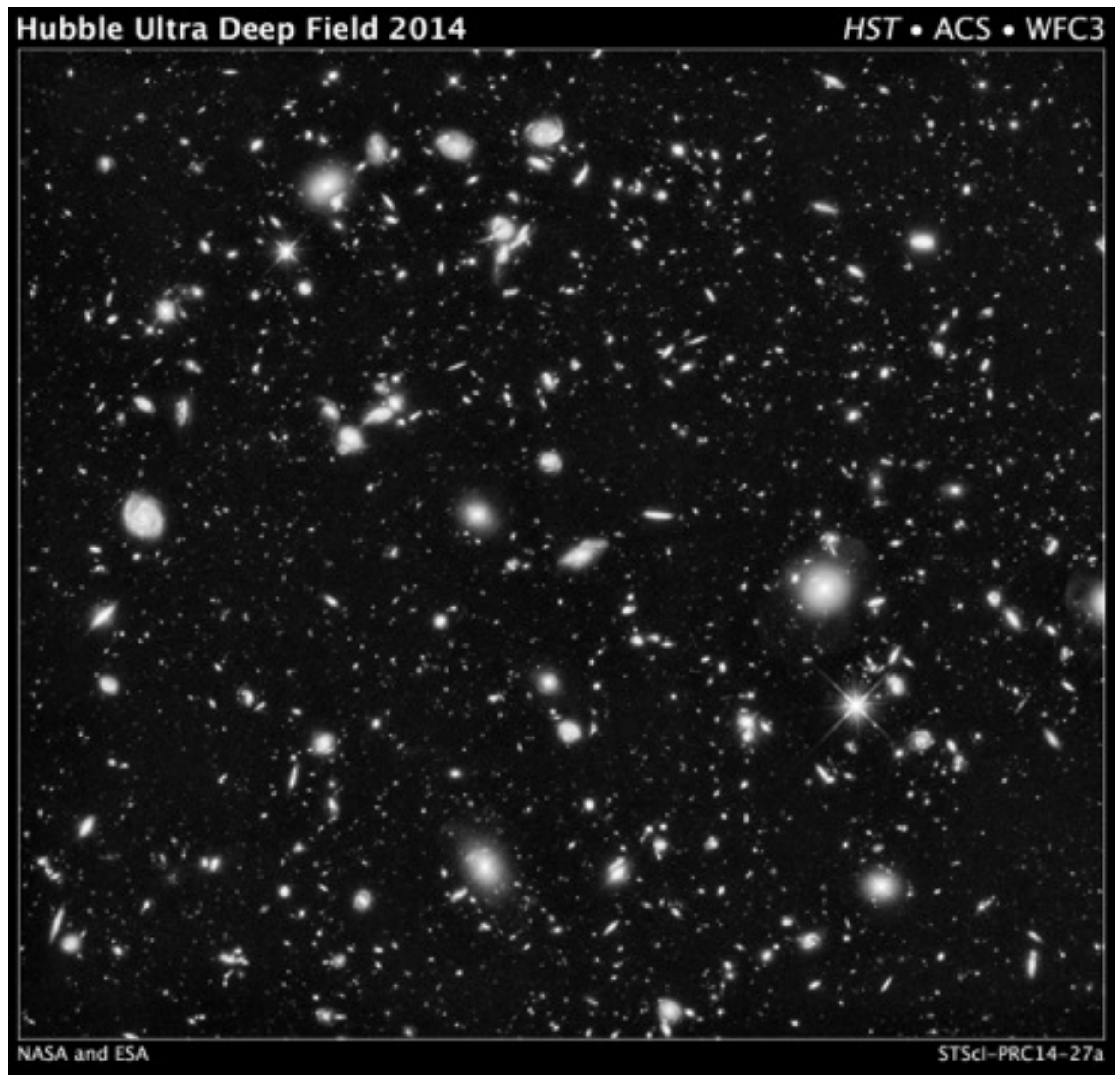
<u>Weak gravitational lensing</u>



- The PSF effect mimics a weak lensing signal
- Data driven approach

# Data analysis challenges



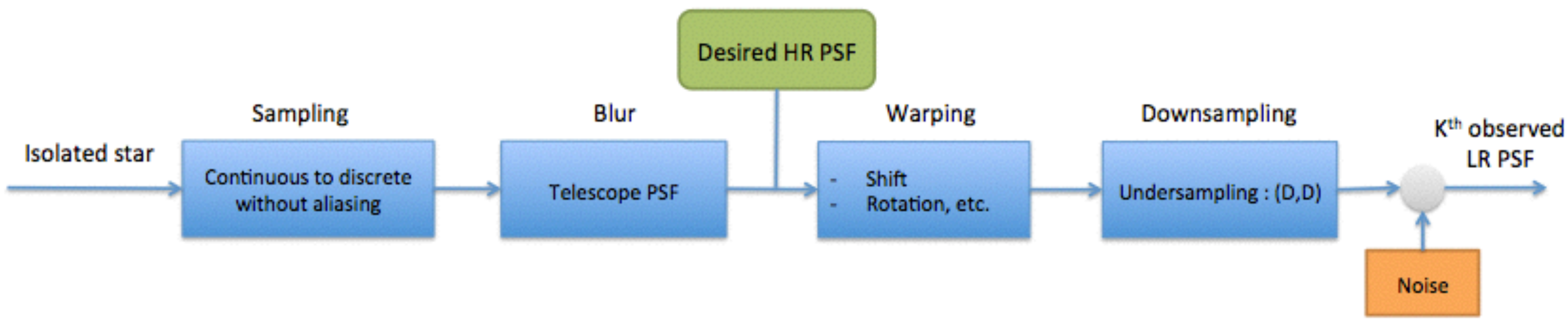
# Data analysis challenges

# Data analysis challenges

- PSF estimation:
  - undersampling
  - spatial and temporal variations
  - wavelength dependency
- Unseen galaxy shape recovery

# PSFs field SR (in prep)

## Classical observation model




$$\mathbf{y}_k = \mathbf{M}_k \mathbf{x} + \mathbf{n}_k,\ k = 1..n$$

$\mathbf{y}_k$: $k^{th}$ low resolution image
$\mathbf{M}_k$: warping and downsampling operator
$\mathbf{x}$: well resolved image
$\mathbf{n}_k$: gaussian noise

# PSFs field SR (in prep)

Field observation model

$$\mathbf{y}_k = \mathbf{M}_k \mathbf{x}_k + \mathbf{n}_k,\ k = 1..n$$

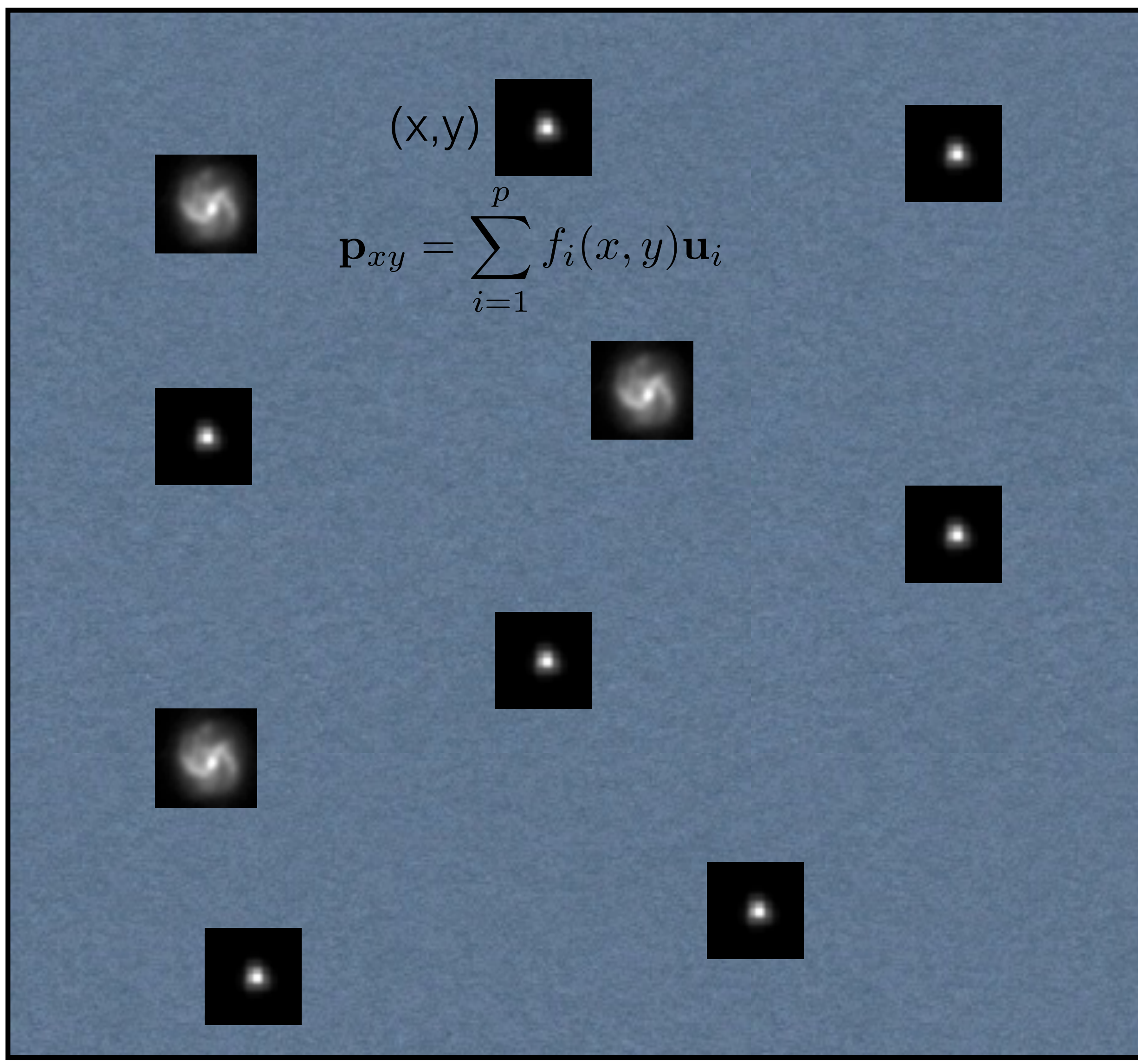
$$\mathbf{M}_k :\ p \times d^2 p$$

Recover n well sampled different PSFs from n low resolution observations

# PSFs field SR

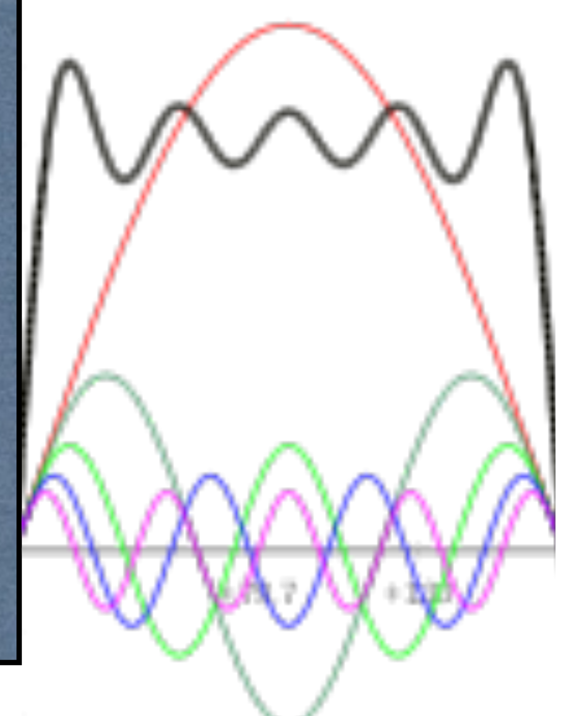
Well resolved PSFs => p pixels

$(\mathbf{u}_i)_{1 \leq i \leq p} \in (\mathbb{R}^p)^p$ $\mathbf{u}_i$ = Images




Expansion of the functions $f_i$ onto a spatial frequencies function basis

$$f_i(x,y) = \sum_{k=0}^{\infty} c_{ik} a_k(x,y)$$



$$\mathbf{s}_k = \sum_{i=1}^{p} c_{ik} \mathbf{u}_i$$

$$\mathbf{p}_{xy} = \sum_{k=0}^{\infty} a_k(x,y) \mathbf{s}_k$$

# PSFs field SR

Method

$$\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_n]$$

$$\mathbf{x}_k = \sum_{l=1}^{r} a_{lk} \mathbf{s}_l \qquad \mathbf{X} = \mathbf{S}\mathbf{A}$$

Prior informations

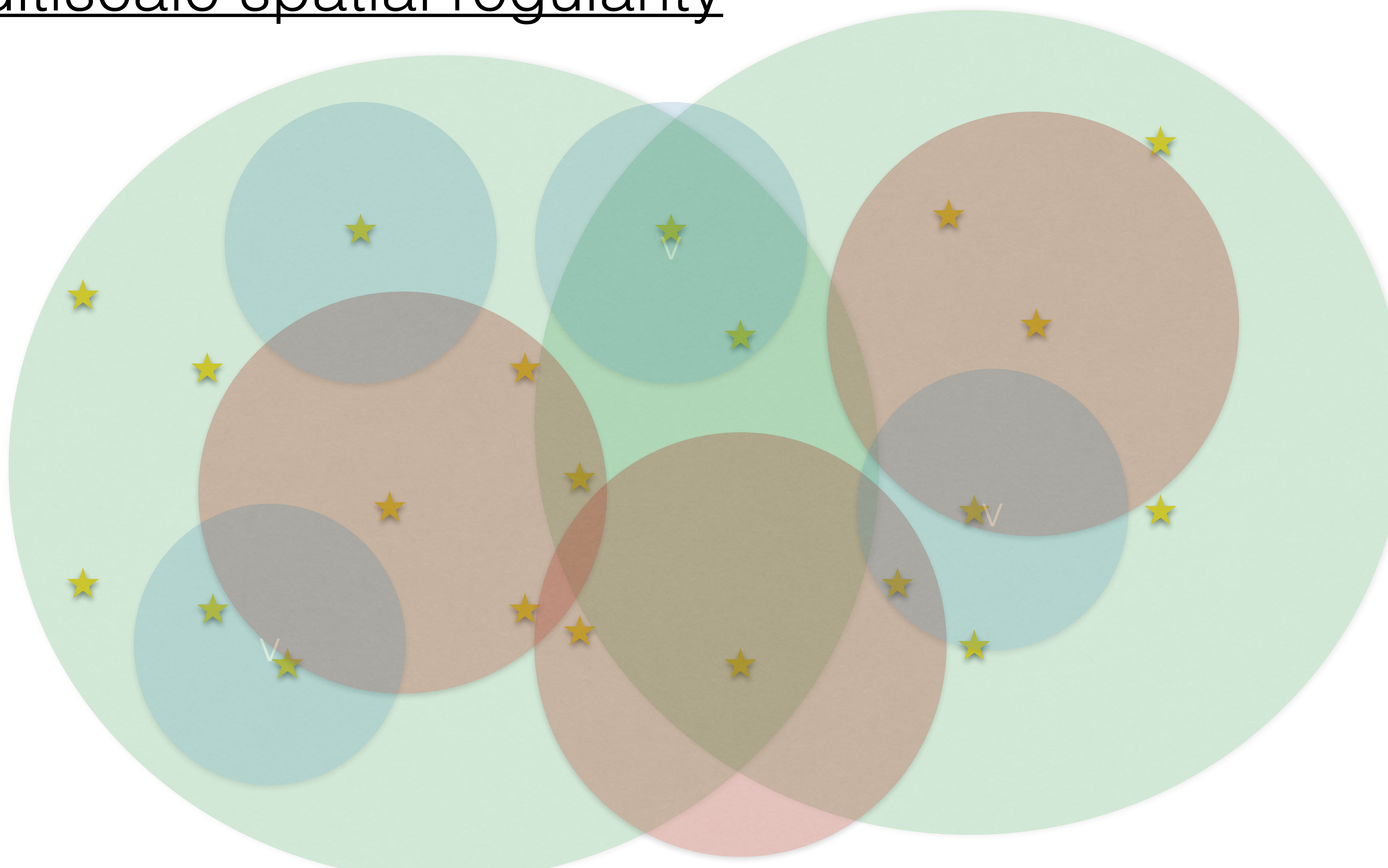
$$\mathbf{S}\mathbf{A} \geq 0$$

Sparse prior on the components

“Frequencies separation” constraint

# PSFs field SR

Multiscale spatial regularity



# PSFs field SR

Multiscale spatial regularity

$$h_l(\mathbf{a}) = \sum_{i=1}^{n} \sum_{j \in knn(i)} f(d(i,j))(a_i - a_j)^2 \ , f(d) = (d_0/d)^{m_l},$$

$$l = 1..r,\ m_1 < m_2 < ... < m_r$$

$$\mathcal{H}(\mathbf{A}) = \sum_{l=1}^{r} h_l(\mathbf{a}_l^L)$$

# PSFs field SR

Optimization problem

$$\min_{\mathbf{A},\mathbf{S}} \sum_{k=1}^{n} \|\mathbf{y}_k - \mathbf{M}_k(\sum_{l=1}^{r} a_{lk}\mathbf{s}_l)\|_2^2 + \mathcal{H}(\mathbf{A}) + \mathcal{G}(\mathbf{S}) \; s.t. \mathbf{SA} \geq 0$$

$$\mathcal{G}(\mathbf{S}) = \sum_{l}^{r} \|w_l \odot \Phi\mathbf{s}_l\|_1$$

Alternate minimization scheme between A and S

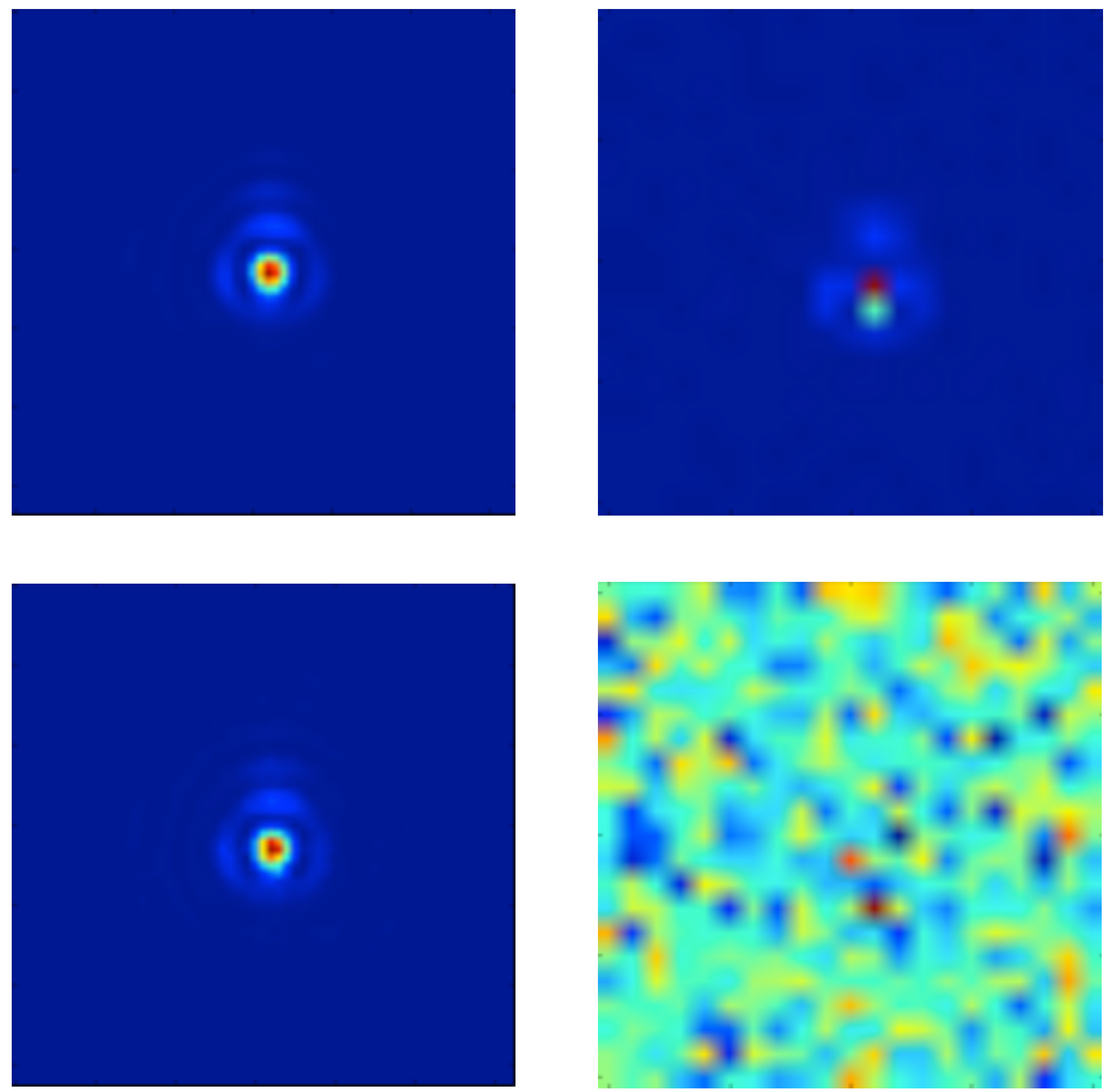
# PSFs field SR

Numerical experiment

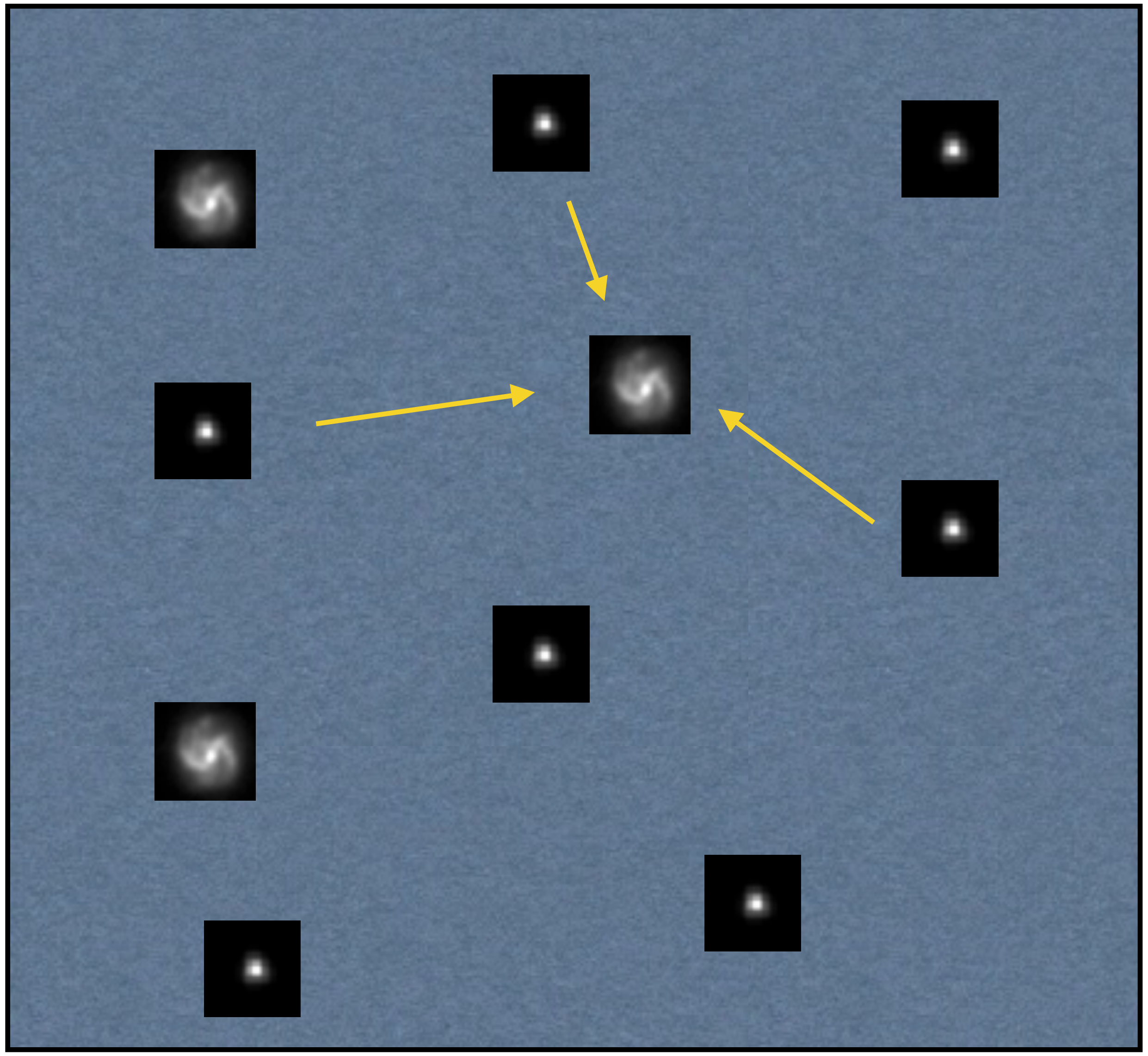
- 50 noisy different PSFs at Euclid resolution
- 50 upsampled PSFs with a factor 3 in lines and columns

# PSFs field SR

Numerical experiment



# PSFs interpolation



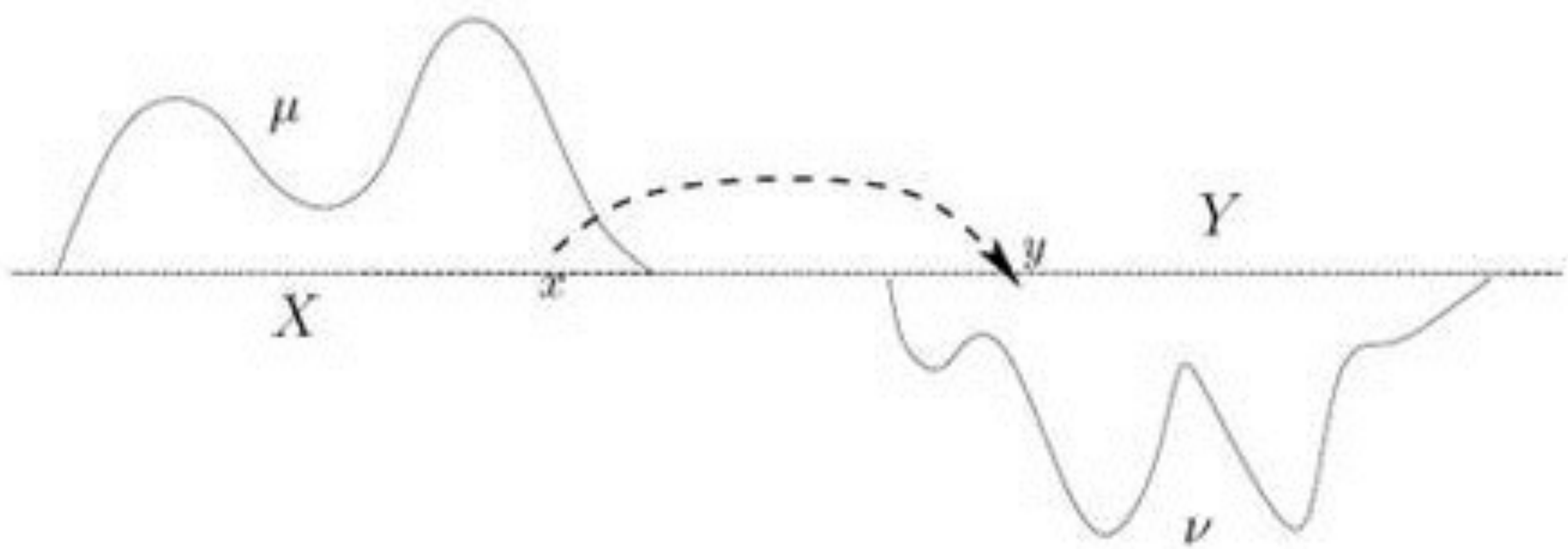
(Almost) No information on the PSFs at the galaxies locations

$$PSF = \underset{\mathbf{x}}{argmin} \sum_{i \in neighb} w_i d(\mathbf{x}, PSF_i)$$

# PSFs interpolation

<u>Optimal transport</u>

G. Monge, “Mémoire sur la théorie des déblais et des remblais”, 1781



# PSFs interpolation

Monge-Kantorovich formulation

$$\min_{\Gamma} \sum_{i,j} c(i,j)\Gamma_{i\to j} \qquad s.t. \ \sum_{i} \Gamma_{i\to j} = \nu_j$$

$$\sum_{j} \Gamma_{i\to j} = \nu_i$$

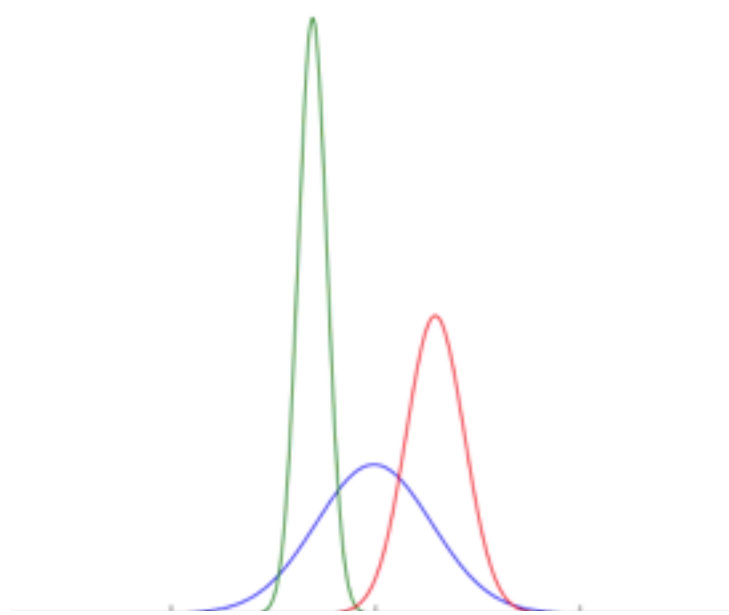
$$\Gamma_{i\to j} \geq 0$$

- Transport distance
- Entropic regularization

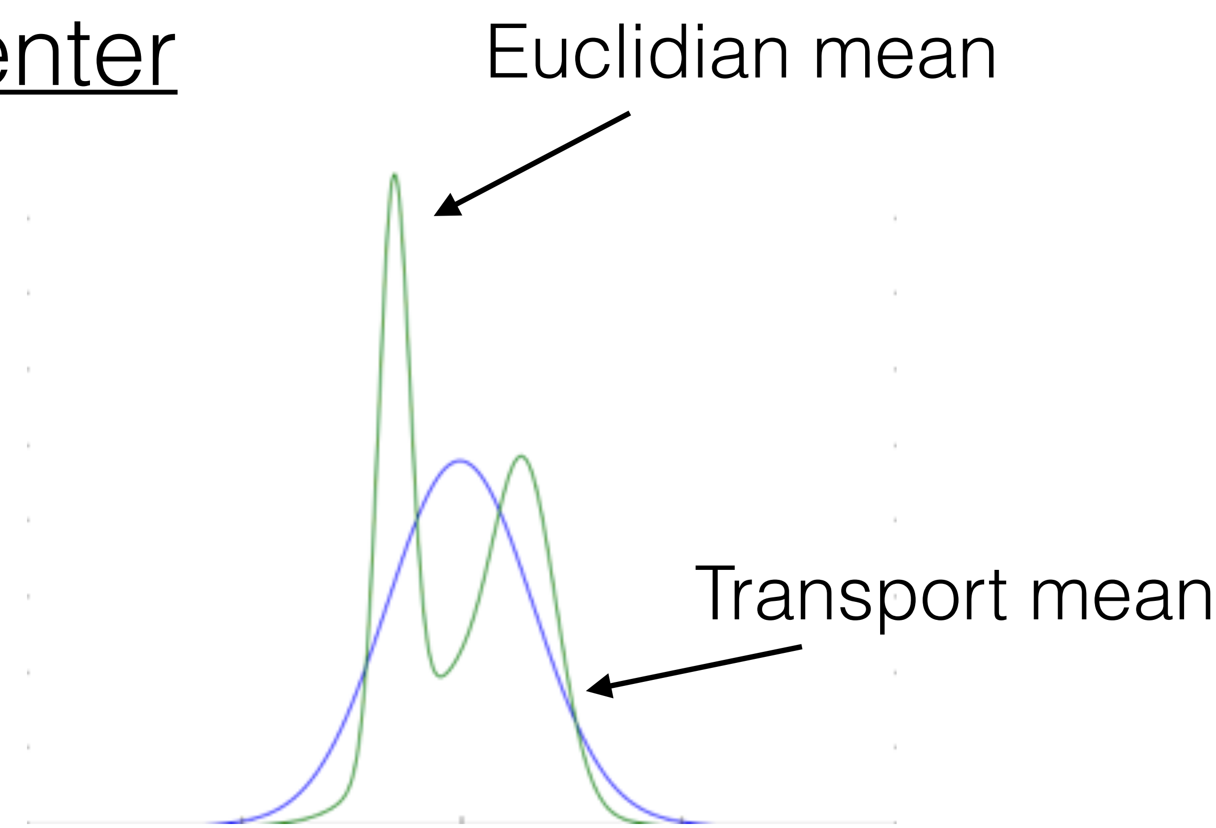
allowing a fast solving (M. Cuturi 2013)

# PSFs interpolation

<u>Barycenter</u>



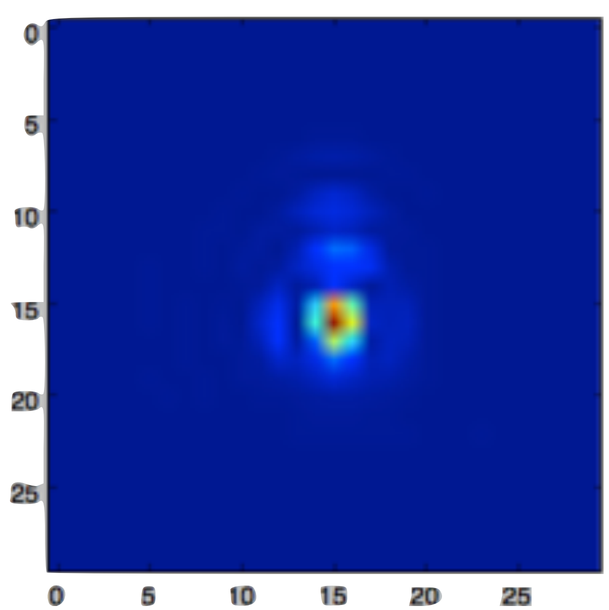
Three gaussians normalized in l1 norm



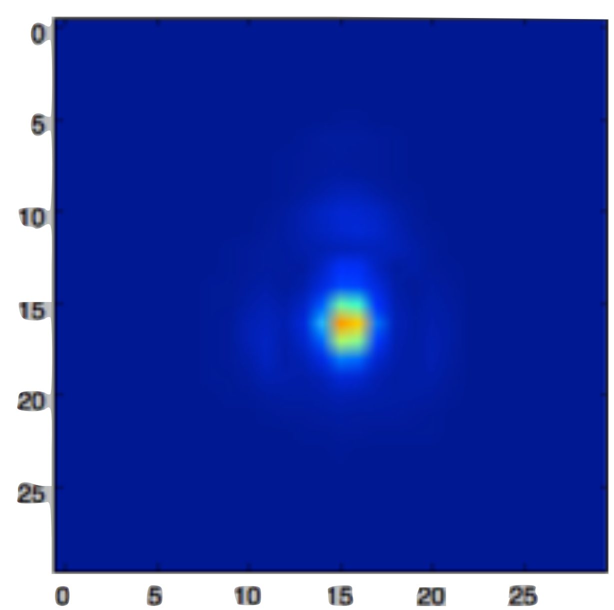

- Plausible average distribution in presence of shifts and changing of shape

# PSFs interpolation

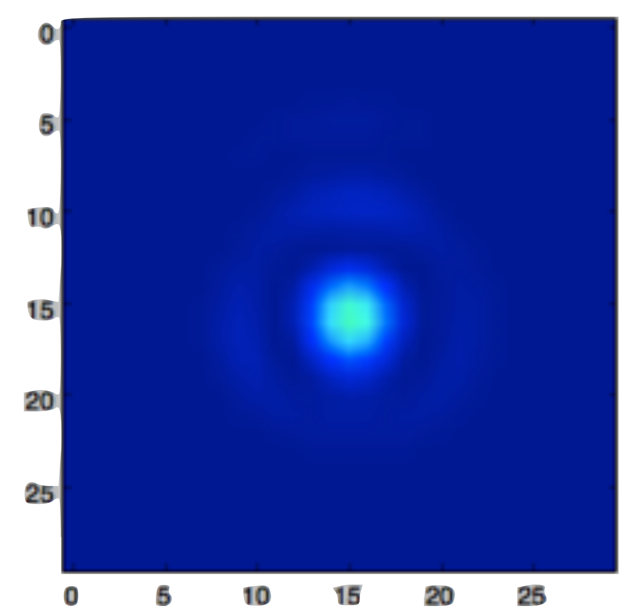
Averaging size varying PSFs




PSF at 550nm




Interpolated PSF




PSF at 900nm

# Conclusions

- Robust super-resolution method for smooth PSFs fields
- d=1 => dimension reduction method separating decomposing the field into high and low frequencies features
- the transport metric: convenient for the interpolation task