Missing Data in turbulent flows

Reconstructing turbulent flows from partial measurements and statistics

Bérengère Podvin

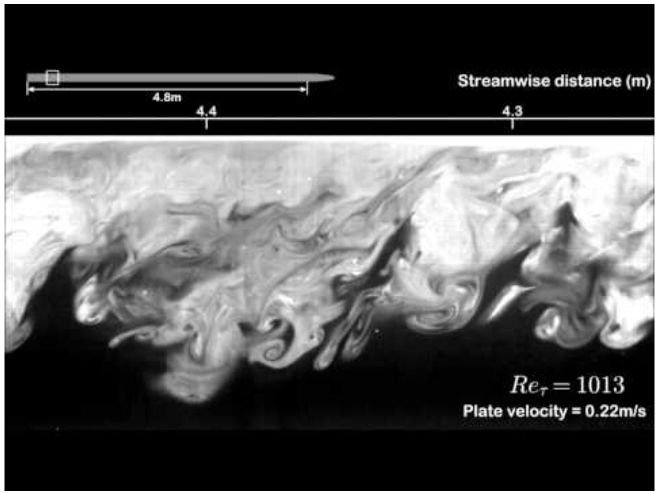
Yann Fraigneau, Francois Lusseyran, Luc Pastur

LIMSI-CNRS, Orsay

Outline

- Turbulence and patterns
- Proper Orthogonal Decomposition
- Application to 3D reconstruction of cavity flow
- Application to channel flow

Turbulence


Streamwise distance (m)
4.8m
4.4
4.3
$Re_\tau = 1013$
Plate velocity = 0.22m/s

A few words about turbulence

- 3D, randomness, vorticity, mixing…
- Dissipation weak (large Reynolds and Rayleigh numbers) but crucial
- Large spectrum of spatial and temporal scales
- “Chaotic”, “Intermittent” Dynamics
- Presence of organized patterns: coherent structures

Missing data

- Spatio-temporal complexity cannot be entirely captured:
- in experiments:
  - **few sensors**
  - intruding measures
  - noisy signals
- in numerical simulations:
  - **spatial scale resolution** (both large and small)
  - “short” integration times

The organization of turbulence

- Complex inter-dependence between all scales of the flow

➔ A structure-based representation of fluid flows

- Spatial patterns: Proper Orthogonal Decomposition (Principal Component Analysis, Karhunen-Loève Decomposition)

Proper Orthogonal Decomposition (POD)

Separation

$$\underline{u}(\underline{x},t) = \sum_n a^n(t)\underline{\phi}^n(\underline{x})$$

Optimization

$$Max \frac{< \underline{u}(\underline{x},t).\underline{\phi}(\underline{x}) >}{< \underline{\phi}(\underline{x}).\underline{\phi}(\underline{x}) >}$$

➔Eigenvalue problem

$$\int < \underline{u}(\underline{x},t)\underline{u}(\underline{x}',t) > .\underline{\phi}(\underline{x}')d\underline{x}' = \lambda\underline{\phi}(\underline{x})$$

Orthogonality

$$< a^n(t)a^m(t) >= \delta_{nm}\lambda^n$$

$$\int \underline{\phi}^n(\underline{x}).\underline{\phi}^m(\underline{x})d\underline{x} = \delta_{nm}$$

If ϕ is known, knowledge of u(x,t) ←→ $a^n(t) = \int \underline{u}(\underline{x},t).\underline{\phi}^n(\underline{x})d\underline{x}$

POD: Method of Snapshots

- Sirovich (1987)
- Finite Size of the practical problem: N flow realizations (snapshots)

$$< \underline{u}(\underline{x},t)\underline{u}(\underline{x}',t) >= \frac{1}{N}\sum_{n=1}^{N} \underline{u}(\underline{x},t^n)\underline{u}(\underline{x}',t^n)$$

- If N independent realizations, eigenfunctions can be written in that base

$$\underline{\phi}^p(\underline{x}) = \sum_{n=1}^{N} A_p^n \underline{u}(\underline{x},t^n)$$

- Equivalent eigenvalue problem

$$\int < \underline{u}(\underline{x},t)\underline{u}(\underline{x}',t) > .\underline{\phi}^p(\underline{x}')d\underline{x}' = \lambda^p \underline{\phi}^p(\underline{x}) \Leftrightarrow \boxed{\frac{1}{N}\int \underline{u}(\underline{x},t^n).\underline{u}(\underline{x},t^m)d\underline{x}A_p^m = \lambda^p A_p^n}$$

- Determining structure amplitude

$$\underline{a^p(t^n)} = \int \underline{u}(\underline{x},t^n).\underline{\phi}^p(\underline{x})d\underline{x} = \int \underline{u}(\underline{x},t^n).\underline{u}(\underline{x},t^q)A_p^q\, d\underline{x} = \underline{A_p^n}$$

Missing data : Gappy POD (Everson and Sirovich 95)

Consider realization $u(\underline{x}_i,t) = u_i^t \rightarrow h_i^t = \begin{cases} 1 \text{ if information accessible} \\ 0 \text{ if missing information} \end{cases}$

- Case 1 : IF Eigenfunctions are fully known (**full** set of snapshots)

$u(\underline{x},t) = \sum_{n=1} a^n(t)\phi^n(\underline{x})$? Estimate $\tilde{a}^n$ such that $B_{nm}\tilde{a}^m = b_n$

$$B_{mn} = \sum_i \phi^m(\underline{x}_i)\phi^n(\underline{x}_i)h_i^t$$

$$b_n = \sum_i u(\underline{x}_i,t)\phi^n(\underline{x}_i)h_i^t$$

- Case 2: IF Eigenfunctions are not fully known (**incomplete** set of snapshots):

-Solve eigenvalue problems for successive (k) snapshots sets

$$\underline{u}^{(k)}(\underline{x}_i,t^n) = \underline{u}(\underline{x}_i,t^n) \text{ if } h_i^n = 1$$

$$\underline{u}^{(k)}(\underline{x}_i,t^n) = \underline{u}^{(k-1)}(\underline{x}_i,t^n) \text{ if } h_i^n = 0$$

$$\text{where } \begin{cases} \underline{u}^{(k-1)}(\underline{x}_i,t^n) = \sum_{m=1} a^m_{(k-1)}(t^n)\underline{\phi}^m_{(k-1)}(\underline{x}_i) \text{ if } k > 2 \\ \underline{u}^{(1)}(\underline{x}_i,t^n) = \frac{1}{N}\sum_{n=1}^{N}\underline{u}(\underline{x}_i,t^n)h_i^n \end{cases}$$

➔ Determine eigenfunctions fully

➔ Go to Case1

Extended POD (Borée 2002)

Consider N realizations q(x,t) with dim(q)=r and apply Snapshot (if necessary Gappy) POD to p<r components

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_p \end{pmatrix}(\underline{x},t) = \sum_{n=1}^{N} a^n(t) \begin{pmatrix} \phi_1^n \\ \phi_2^n \\ \phi_3^n \\ \vdots \\ \phi_p^n \end{pmatrix}(\underline{x}) \text{ with } \begin{pmatrix} \phi_1^n \\ \phi_2^n \\ \phi_3^n \\ \vdots \\ \phi_p^n \end{pmatrix} = \begin{pmatrix} a^1(t_1) & a^1(t_2)\ldots & a^1(t_n) \\ a^2(t_1) & a^2(t_2)\ldots & a^2(t_n) \\ a^3(t_1) & a^{31}(t_2)\ldots & a^3(t_n) \\ \vdots & \vdots & \vdots \\ a^n(t_1) & a^n(t_2)\ldots & a^n(t_n) \end{pmatrix} \begin{pmatrix} q_1(\underline{x},t_1) \\ q_2(\underline{x},t_2) \\ q_3(\underline{x},t_3) \\ \vdots \\ q_p(\underline{x},t_n) \end{pmatrix}$$

It is possible to define extended structures on the remaining r-p=r' components using

$$\begin{pmatrix} \phi_{p+11}^n \\ \phi_{p+2}^n \\ \phi_{p+3}^n \\ \vdots \\ \phi_r^n \end{pmatrix} = \begin{pmatrix} a^1(t_1) & a^1(t_2)\ldots & a^1(t_n) \\ a^2(t_1) & a^2(t_2)\ldots & a^2(t_n) \\ a^3(t_1) & a^{31}(t_2)\ldots & a^3(t_n) \\ \vdots & \vdots & \vdots \\ a^n(t_1) & a^n(t_2)\ldots & a^n(t_n) \end{pmatrix} \begin{pmatrix} q_{p+1}(\underline{x},t_1) \\ q_{p+2}(\underline{x},t_2) \\ q_{p+3}(\underline{x},t_3) \\ \vdots \\ q_r(\underline{x},t_n) \end{pmatrix}$$

Applications: - Investigate coupling between different components of the realization (e.g p-components: velocity, r' component: pressure)

- Inverse design (e.g p-component: pressure distribution, r'-components: geometry)

Reconstruction from partial instantaneous measurements and full statistics

$\underline{\phi}(\underline{x})$ known, $u(x,t)$ on H (h) $\subset$ $\Omega \Rightarrow u(x,t)$ on Ω?

$$u(\underline{x},t) = \sum_{n=1}^{\infty ?} a^n(t)\phi^n(\underline{x}) \Rightarrow \text{ Determine } a^{\text{n}}(t)\forall n$$

$$B_{nm}\tilde{a}^m = b_n \text{ where } \begin{cases} B_{nm} = \sum_i \underline{\phi}^n(\underline{x}_i).\underline{\phi}^m(\underline{x}_i)h_i^t \\ b_n = \sum_i \underline{u}(\underline{x}_i,t).\underline{\phi}^n(\underline{x}_i)h_i^t \end{cases}$$

But…

Can any flow realization be exactly represented in the finite POD basis?

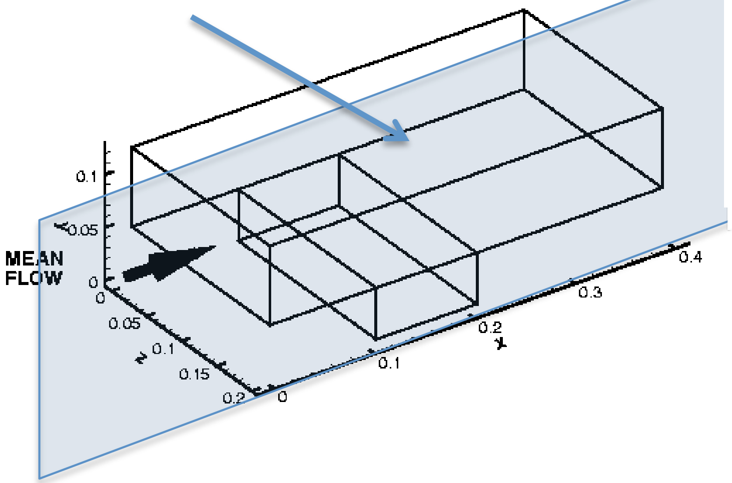
What happens if I consider a smaller number of modes?

How robust is the estimation procedure?

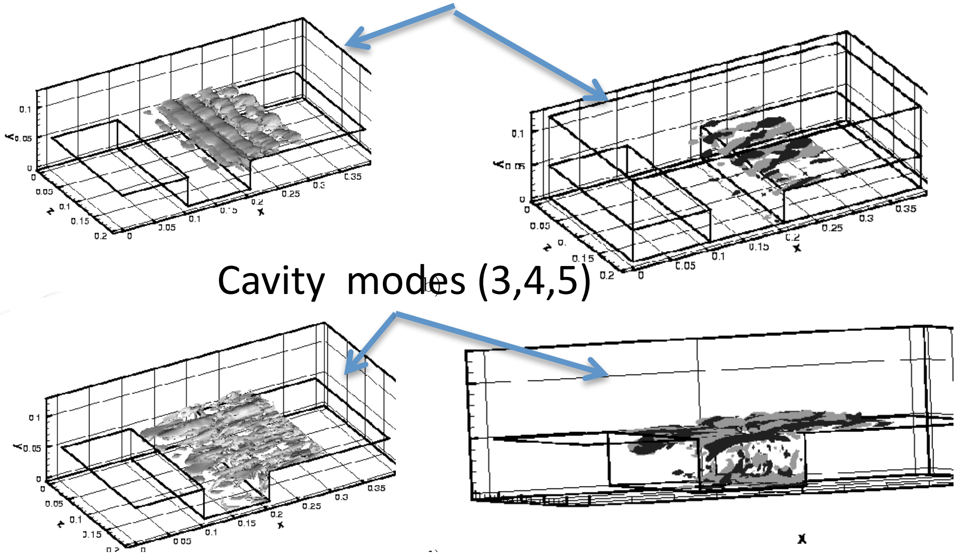
Application: Flow over cavity

Spatial POD eigenfunctions

2-D, 2-C measurement plane



Shear layer modes (1 and 2)



Incompressible flow Re=7500
Code OLORIN (Y. Fraigneau)

Total vorticity

Streamwise vorticity

Goal: Estimate Full 3D flow from 3D,3C POD basis and 2D,2C plane of measurements

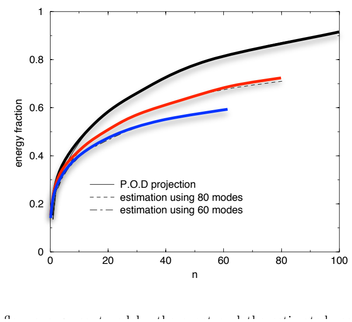
Estimation of POD amplitudes

Podvin et al., JFE 2006

$a^n(t)$

80 modes

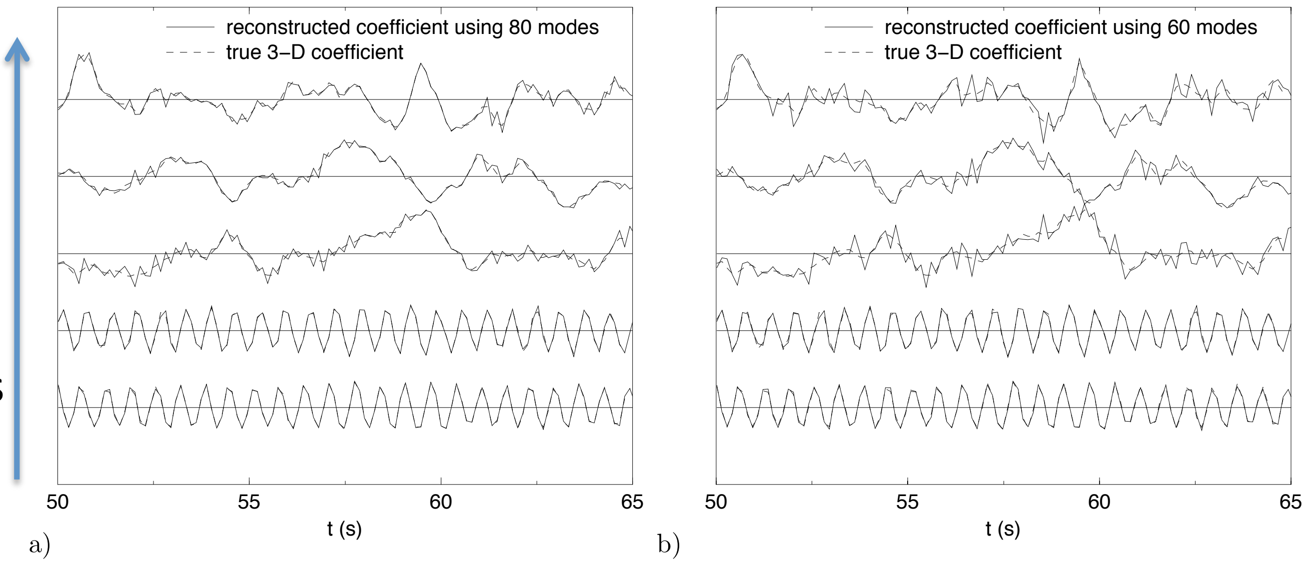
60 modes



Energy fraction

Cavity modes

Shear modes



160 modes (exact)

80 modes (estimation)

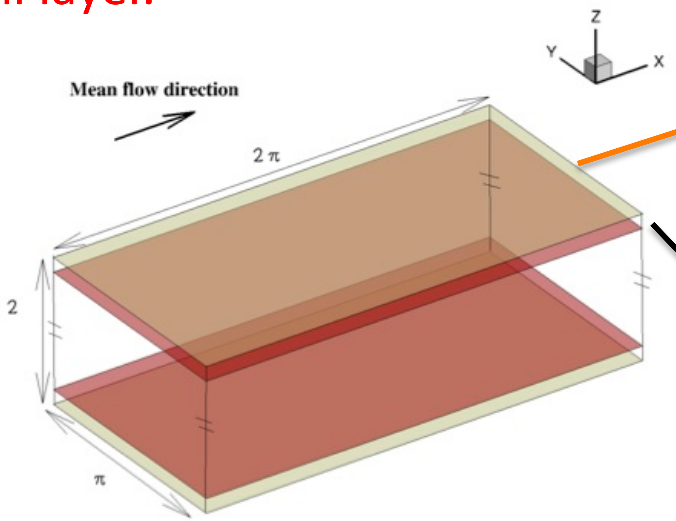
60 modes (estimation)

First POD modes are well recovered but full reconstruction remains a challenge

Application: Synthetic wall boundary conditions for the simulation of wall turbulence

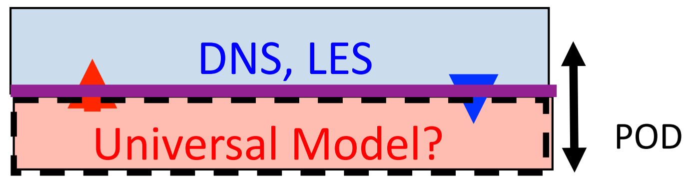
Context:High Reynolds number calculations require fine resolution next to the wall (wall layer) as the dynamics are dominated by small scales

<u>Goal</u>: Simulate turbulence accurately (= get statistics right!) in a restricted domain (H) using a synthetic boundary condition which mimics the top of the wall layer.



99% of the grid points in 10% of BL at Re~10^5

Synthetic Boundary Condition



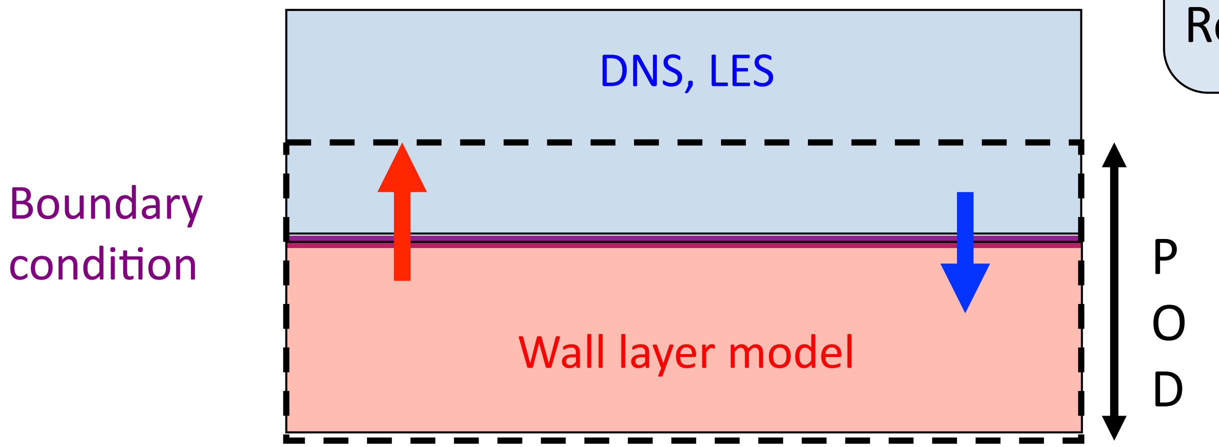
$$\mathrm{Re} = \frac{u_\tau h}{\nu} = 590 \text{ where } u_\tau = \sqrt{\nu \frac{dU}{dy}}_{wall}$$

Reference DNS: SUNFLUIDH code (Y. Fraigneau)

Synthetic boundary condition

DNS/LES Channel flow R

Re=180, 590, 1000



y_{0+}~50,100

dynamically active wall region

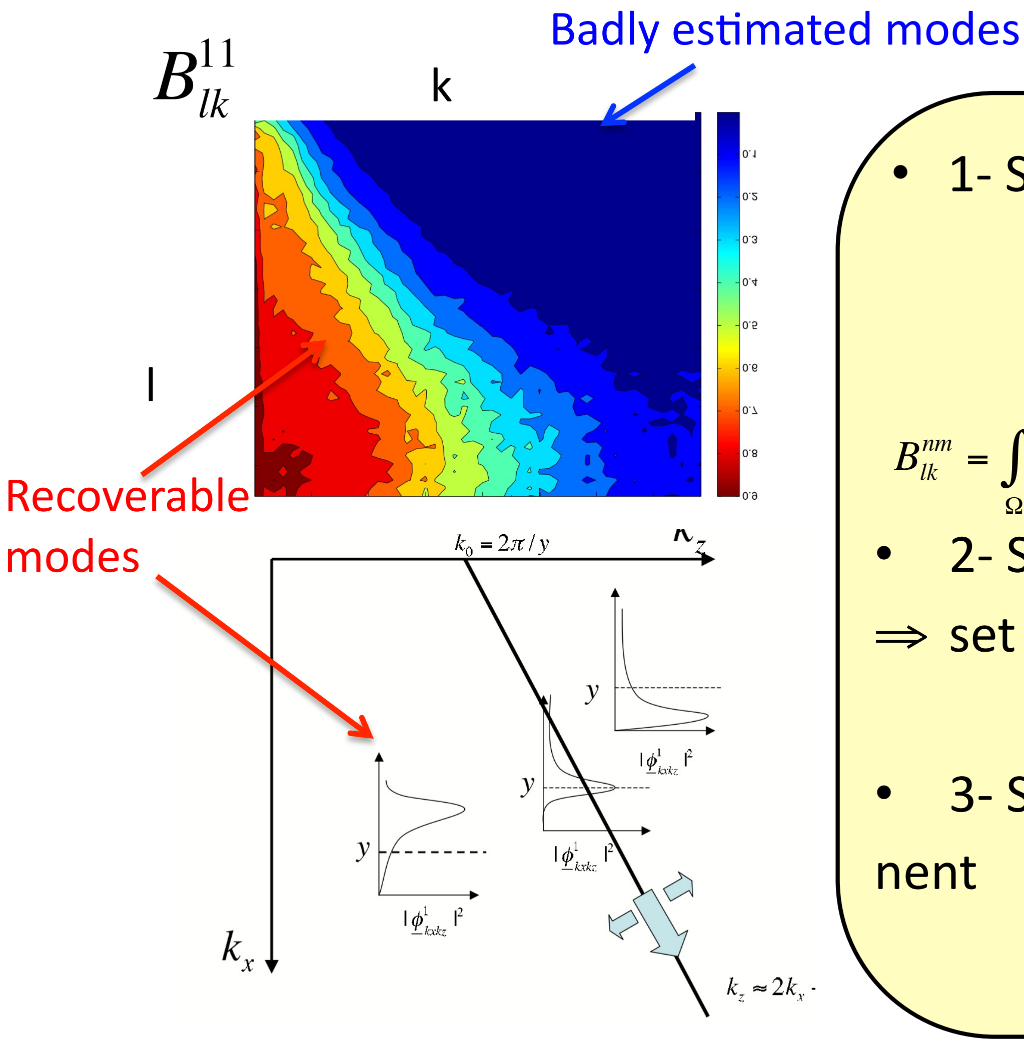
POD-based condition

$$u(x,y,z,t) = \sum_{lkn} a_{lk}^{n}(t)\phi_{lk}^{n}(y)e^{2i\pi lx}e^{2i\pi kz}$$

?

assumed to be known

Feedback estimation (full reconstruction)



- 1- Solve approximate linear system

$$B^{nm}_{lk} a^m_{lk} = b^n_{lk}$$

$$B^{nm}_{lk} = \int_\Omega \underline{\phi}^n_{lk}(\underline{x}) \underline{\phi}^{m*}_{lk}(\underline{x}) d\underline{x} \qquad b^n_{lk} = \int_{\Omega-\Omega_1} \underline{u}(\underline{x},t) \underline{\phi}^n_{lk}(\underline{x}) d\underline{x}$$

- 2- Select "recoverable" modes

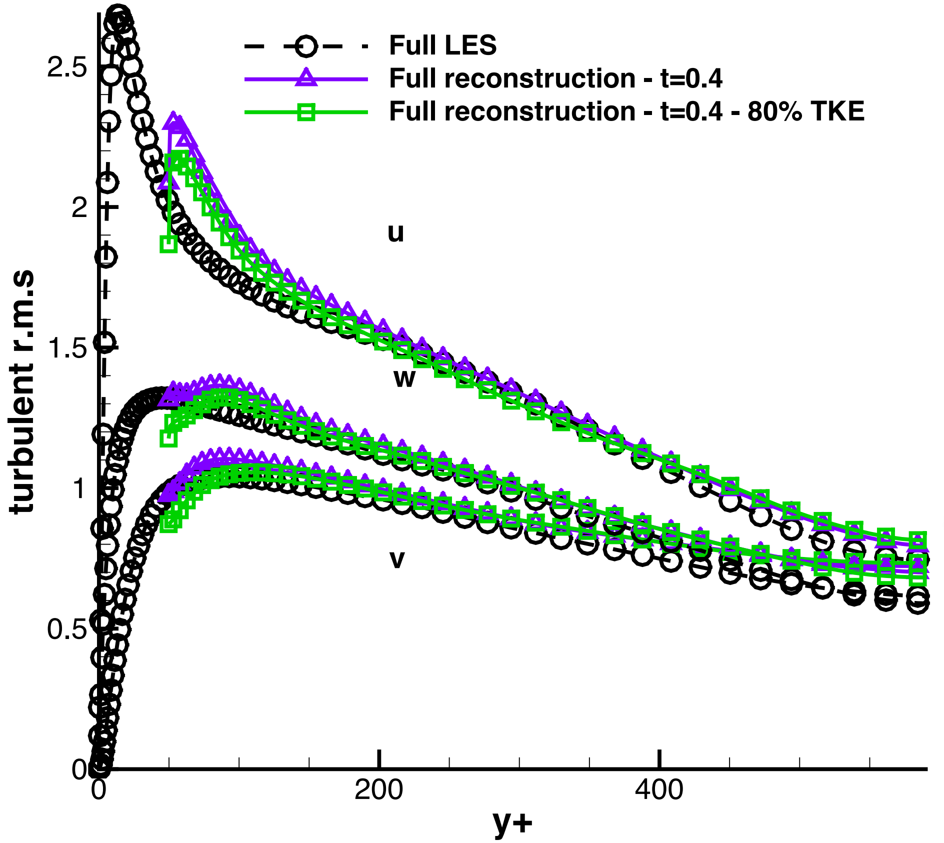
$\Rightarrow$ set $a^n_{lk} = 0$ when $B^{nn}_{lk} < t, \quad t \approx B^{nn}_{l\,0.1k_c}$

- 3- Set L^2 norm of each fluctuating compo-nent

$$\int_{xz} u_i^2 dxdz = C_i^0$$

Full reconstruction Re=590

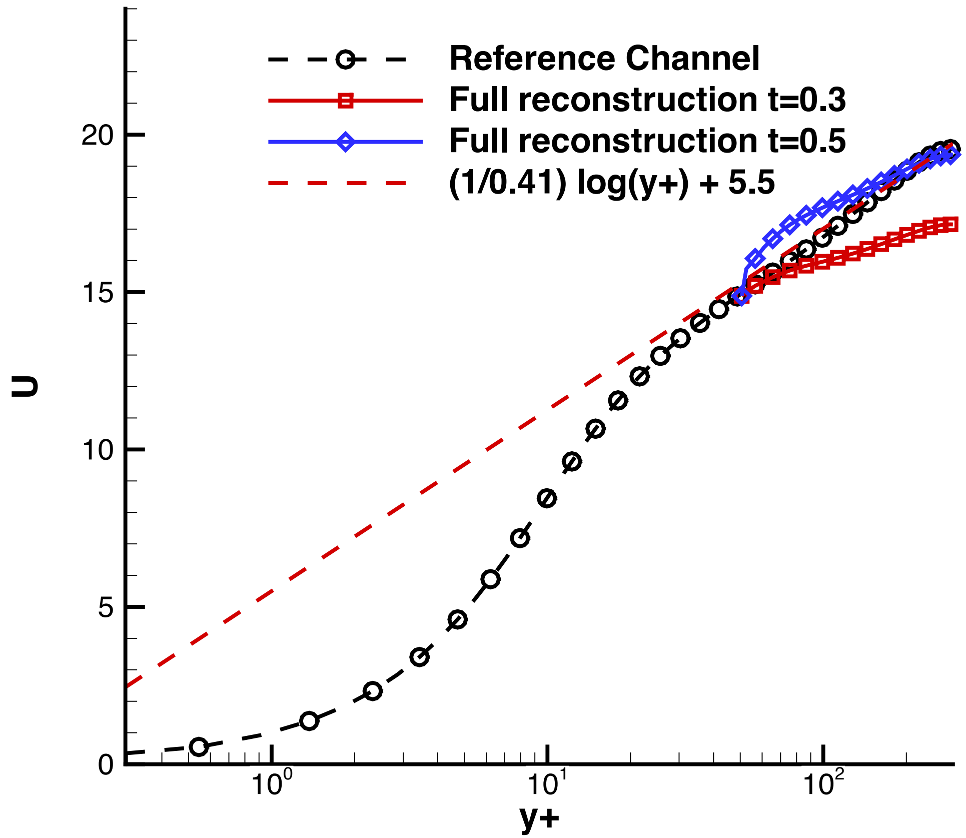
Little sensitivity to energy level C_i^0



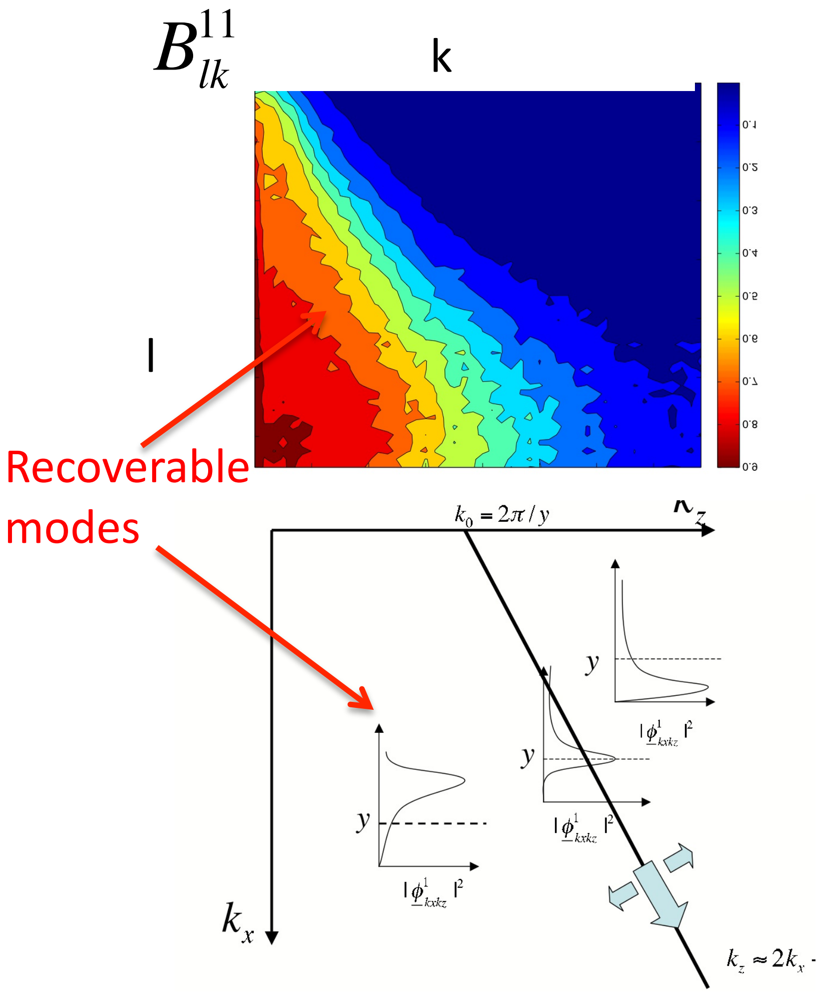
Slight discrepancy at channel center

Full reconstruction (Re=590)

Sensitivity to exact value of threshold



Feedback estimation (phase reconstruction)



- 1- Estimate phase of POD mode with constant modulus

$$a^m_{lk} = (\lambda^n_{lk})^{1/2} Arg[b^n_{lk}]$$

$$\lambda^n_{lk} = <|\underline{a}^n_{lk}(t)|^2>$$

$$b^n_{lk} = \int_{\Omega-\Omega_1} \underline{u}(\underline{x},t)\underline{\phi}^n_{lk}(\underline{x})d\underline{x}$$

- 2- Select "recoverable" modes

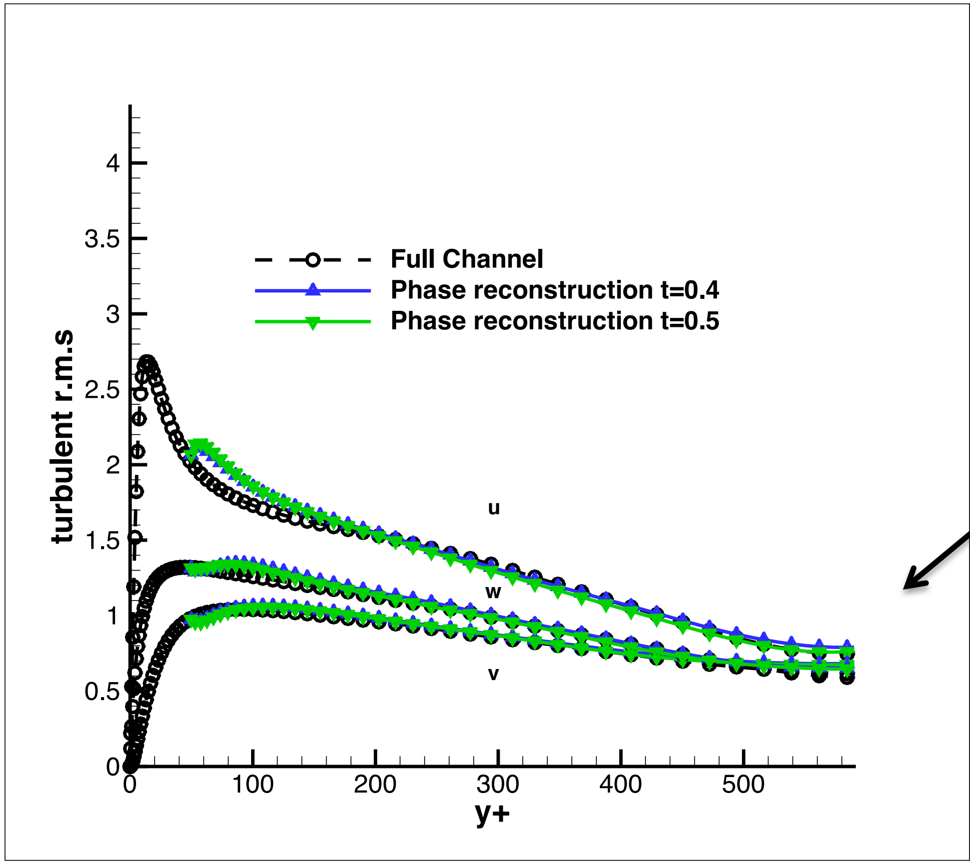
$\Rightarrow$ set $a^n_{lk} = 0$ when $B^{nn}_{lk} < t, \quad t \approx B^{nn}_{l\,0.1k_c}$

- 3- Set L^2 norm of each fluctuating component

$$\int_{xz} u_i^2 dxdz = C_i^0$$

Phase reconstruction Re=590

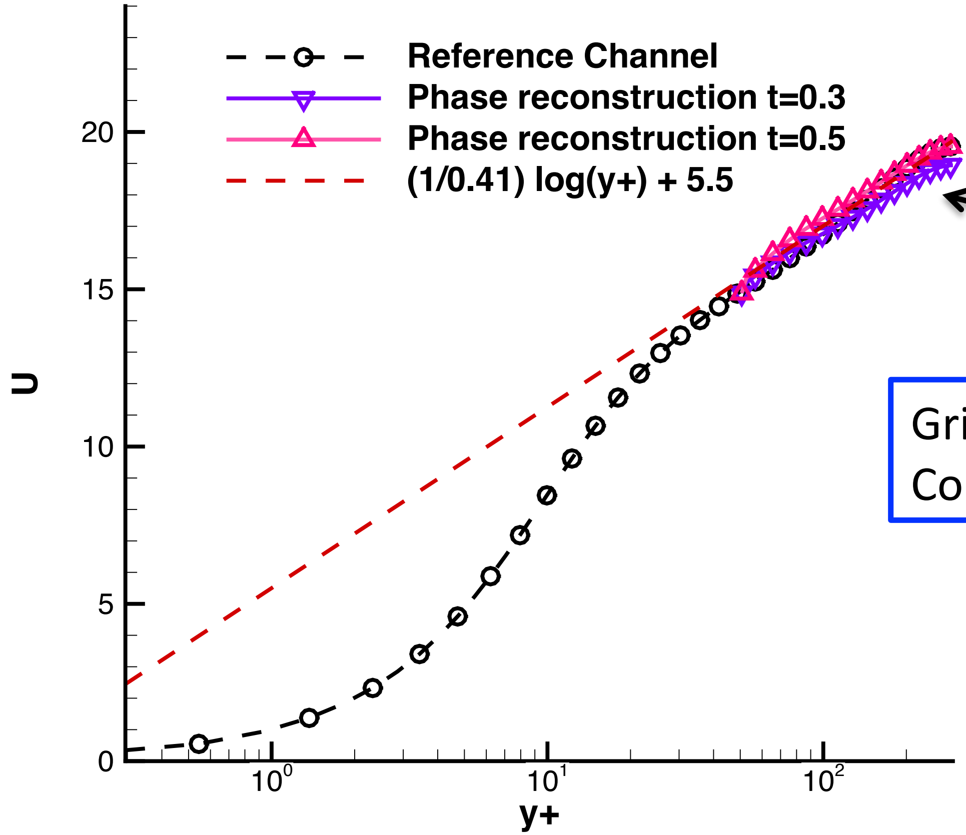
Little sensitivity to exact value of threshold



Less discrepancy at channel center

Phase reconstruction Re=590

Little sensitivity to exact value of threshold



Correct slope

Grid resolution -50%
Computational cost +15%

Statistics are improved with time-dependent phase reconstruction.

Conclusion

- Missing data in turbulence can be recovered with stochastic tools (POD, Gappy POD)
- 2 examples for turbulent flows:
  - Reconstruction of 3D structures from 2D measurements
  - Efficient Simulation of channel flow
- Mathematical tools need to be tailored to flow physics (effect of small scales in turbulence)

References

- Turbulent and the dynamics of coherent structures, L. Sirovich, Quart. Appl. Math., 1987
- The Karhunen-Loève Procedure for Gappy Data, R. Everson, L. Sirovich, J. Opt Soc. Am A, 1995
- Extended proper orthogonal decomposition: a tool to analyse correlated events in turbulent flows, J. Borée, Experiments in Fluids 2003
- "Aerodynamic Data Reconstruction and Inverse Design Using Proper Orthogonal Decomposition", T. Bui-Thanh, M. Damodaran, and K. E. Willcox, AIAA Journal, Vol. 42, No. 8 (2004), pp. 1505-1516.
- "A reconstruction method for the flow past an open cavity", B. Podvin, Y. Fraigneau, F. Lusseyran and P. Gougat, Journal of Fluids Engineering, 2006, 128, 531
- On self-similarity in the inner layer of a turbulent channel flow, B. Podvin, Y. Fraigneau, J.P. Laval, J. Jouanguy, Journal of Fluids Engineering 2010.
- Synthetic wall boundary condition for the direct numerical simulation of wall turbulence, B. Podvin, Y. Fraigneau, Journal of Turbulence 2011, 12(4), 1-26.
- POD-based wall boundary conditions for the numerical simulation of turbulent channel flows, B. Podvin, Y. Fraigneau, Journal of Turbulence 2014, 15, 3, 141-175